

# Mathematical optimization for offshore wind farm design



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# Myself



Double degree in **Automation Engineering** from Padova University (Italy) and Aalborg University (Denmark)

Industrial **PhD in Operational Research**: Collaboration between Technical University of Denmark (DTU Management) and Vattenfall BA Wind

**Now Lead Engineer** in Operations Research Vattenfall BA Wind



# Awards

- 2020 YoungWomen4OR
- 2020 Winner of BIRD ( Best young Italian Researcher in Denmark)
- 2019 Glover-Klingman prize 2018
- 2019 Winner of the EURO Doctoral Dissertation Award (EDDA)
- 2019 Finalist for the INFORMS Franz Edelman Award
- 2019 Erhvervsforskerprisen (Best Industrial Ph.D.) from Innovation Fund Denmark
- 2018 AIRO Best Application Paper Award 2018
- 2018 Finalist for EURO Excellence in Practice Award
- 2017 Best student paper award at ICORES

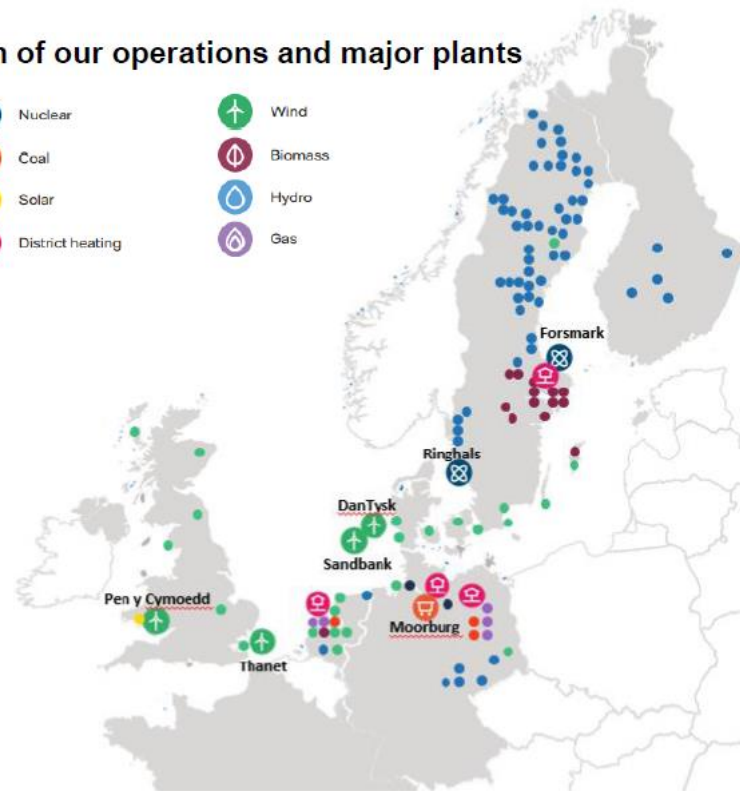


# Vattenfall

## Key data

- One of Europe's largest producers of electricity and heat
- Owned by the Swedish state
- Main products: electricity, heat, gas and energy services
- Main markets: Sweden, Germany, Netherlands, Denmark and the UK
- About 20,000 employees

## Location of our operations and major plants





**Fossil-free within  
one generation**

# Strong competition



“Innovation and competition  
are making sustainable energy  
cheaper and cheaper, and  
much faster than expected  
too.”

*Eric Wiebes - Dutch Economic Affairs and  
Climate Minister, 2018*

# OR for more competitive bids

OR-based tools for

- Wind farm layout optimization
- Cable routing optimization



*M. Fischetti, and D. Pisinger (2018). Mathematical Optimization for offshore wind farm design: an overview. Business and Information Systems Engineering.*

*M. Fischetti (2018) Mathematical Programming Models and Algorithms for Offshore Wind Park Design, PhD thesis, DTU Management & Vattenfall*

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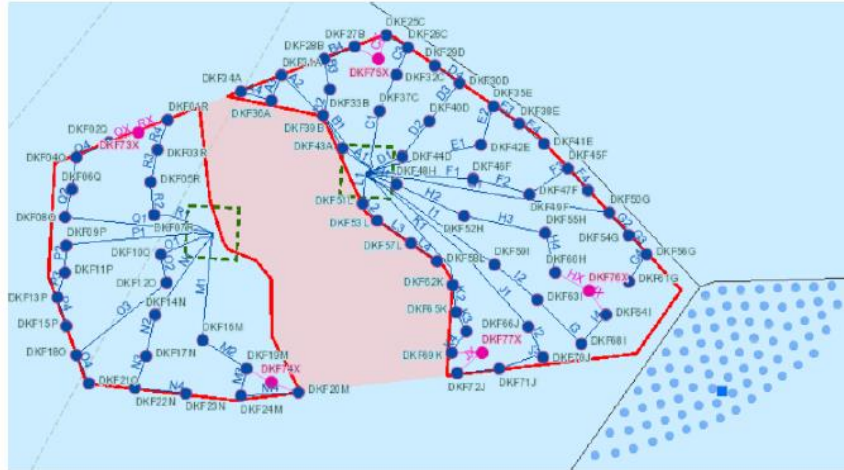
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OR-based tools for

- Wind farm layout optimization
- **Cable routing optimization**



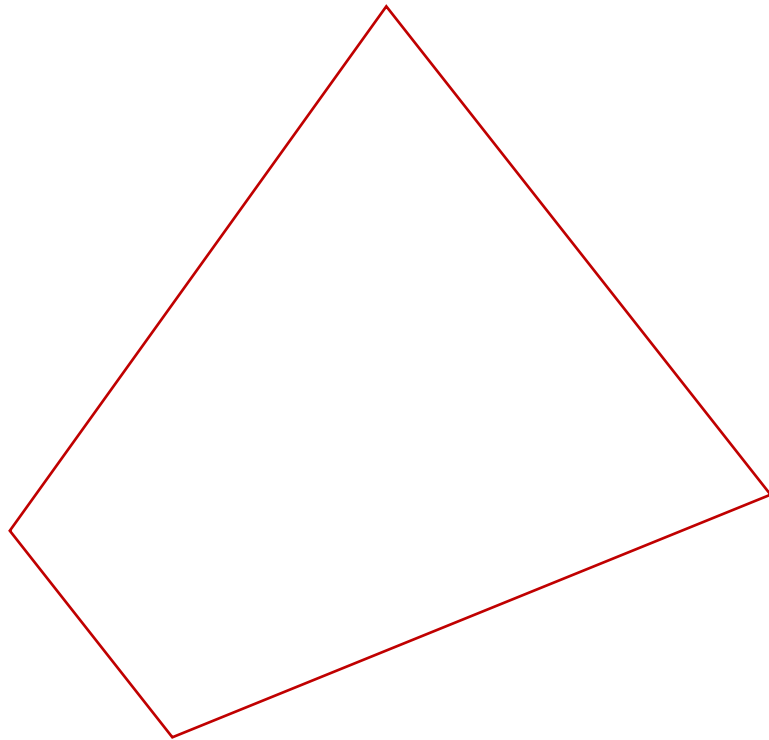
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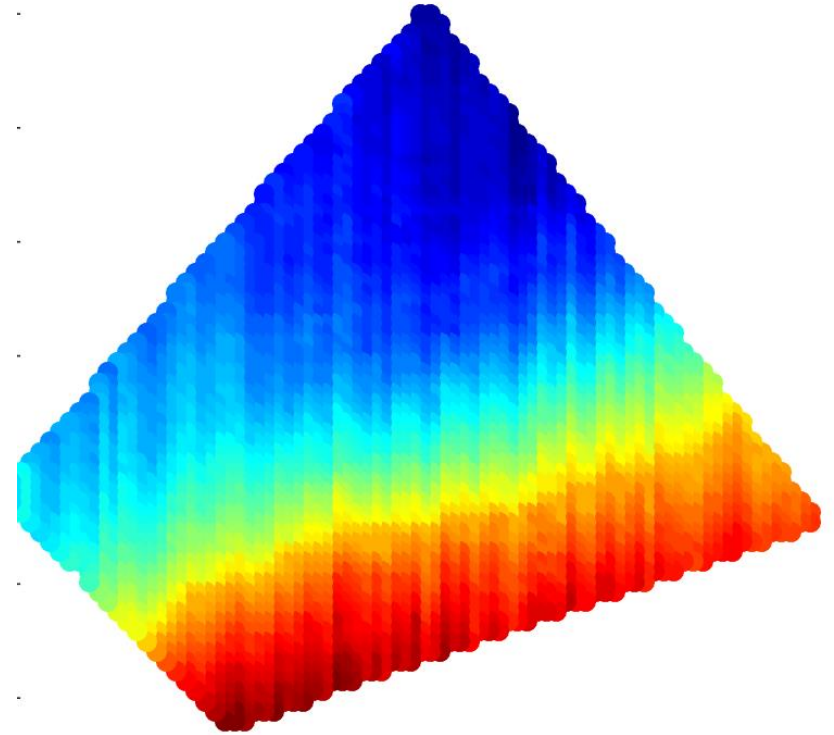
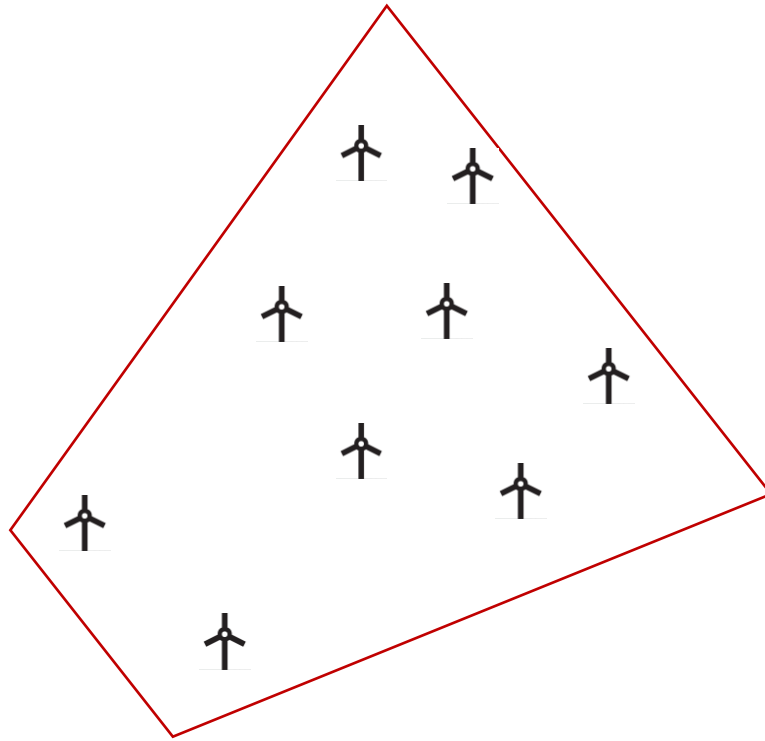
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# Turbine layout optimization



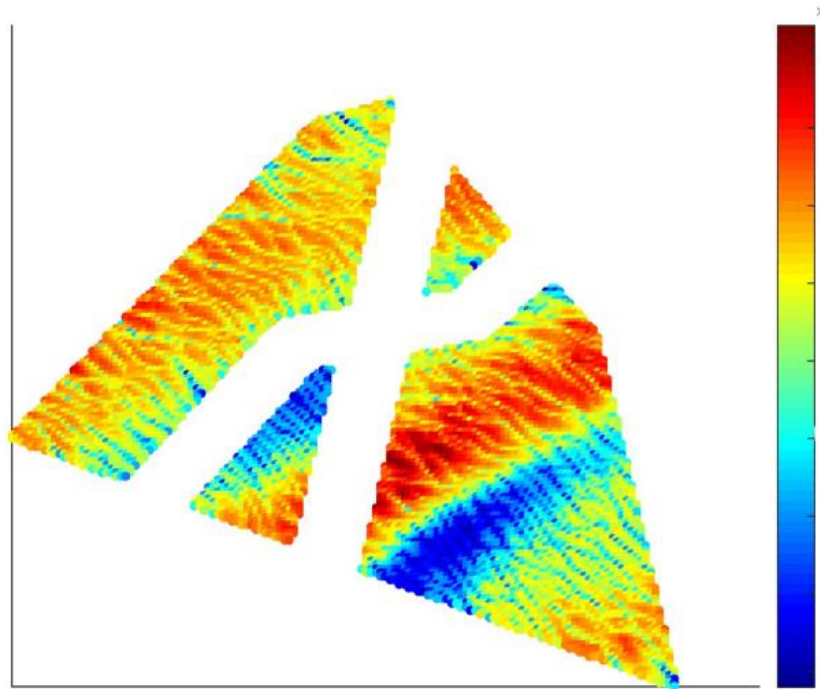






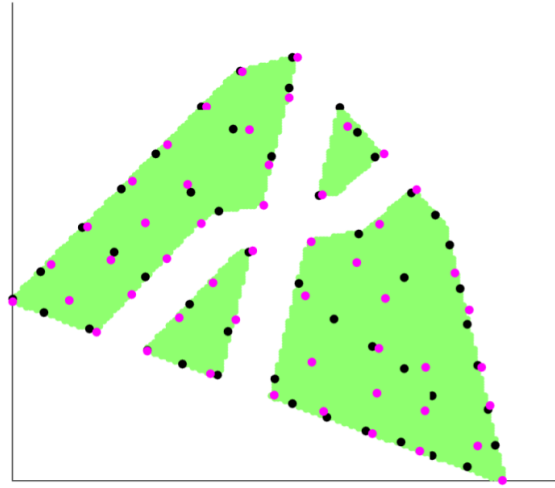
*Martina Fischetti, and Michele Monaci, Proximity search heuristics for wind farm optimal layout, Journal of Heuristics (2016), Volume 22, Issue 4, pp 459–474*

# Impact in practice



Area available - Cost map

# Impact in practice

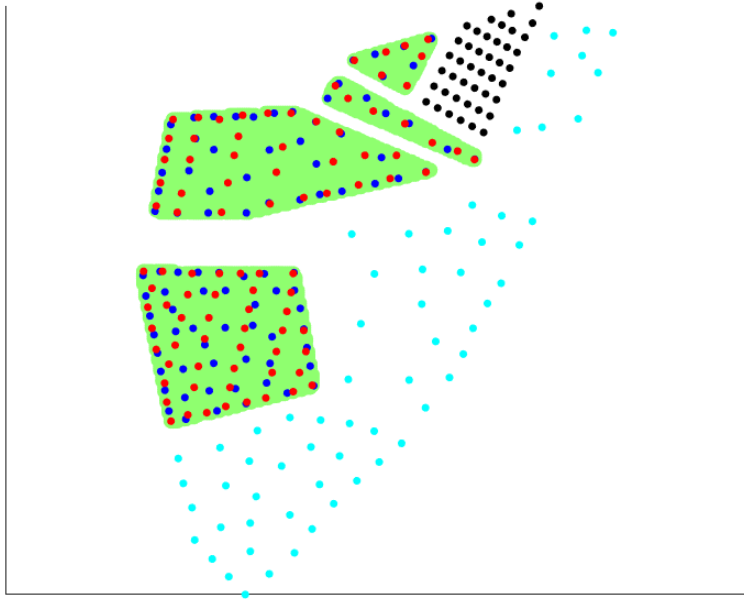


Optimized layout considering wake effect and costs of foundations (black) versus "manual" layout (pink)

Company experts certified that our layout allows for an extra 0.28% production while decreasing cost of foundations of more than 10M euro.

All in all, they estimated an increased income of **12.6Meuro in 25 years.**

# Impact in practice



Our layout (red) allowed for a higher annual energy production: this difference has been evaluated to be worth **10.2 M euro** (Net Present Value) over the park lifetime.

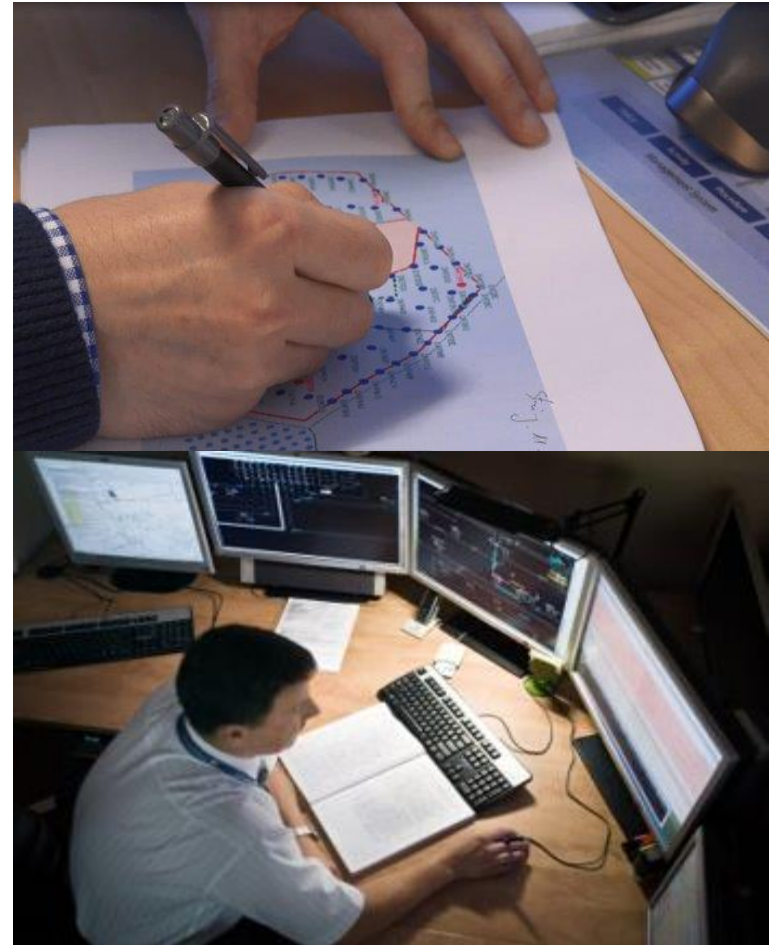
*M. Fischetti, J. R. Kristoffersen, T. Hjort, M. Monaci, and D. Pisinger. Vattenfall optimizes offshore wind farm design. INFORMS Journal on Applied Analytics (2019)*



# A new data-driven process

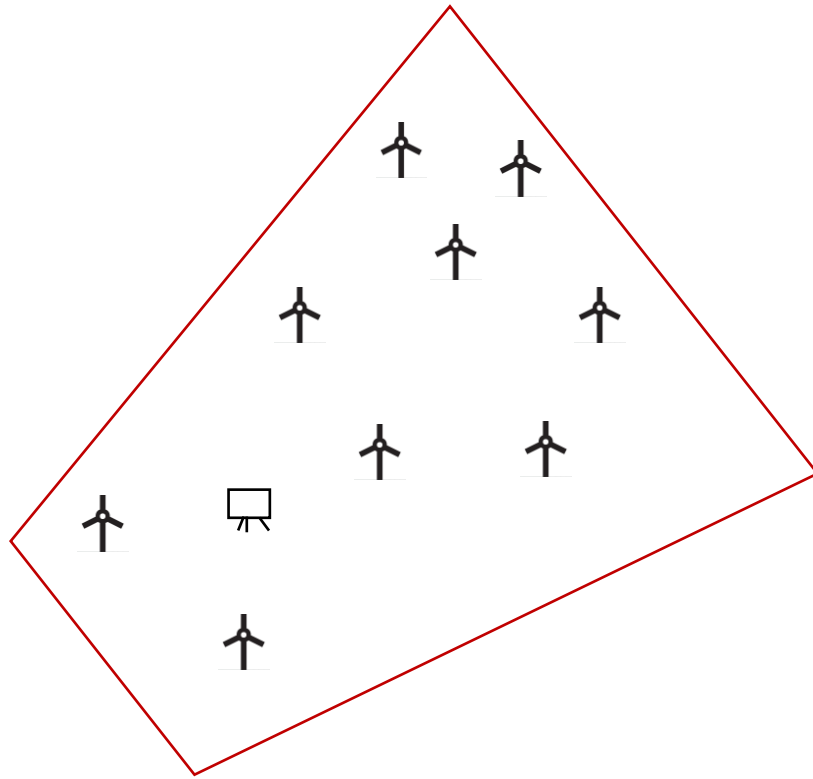
- Big savings
- Faster
- Streamlined & standardized process
- What-if analyses

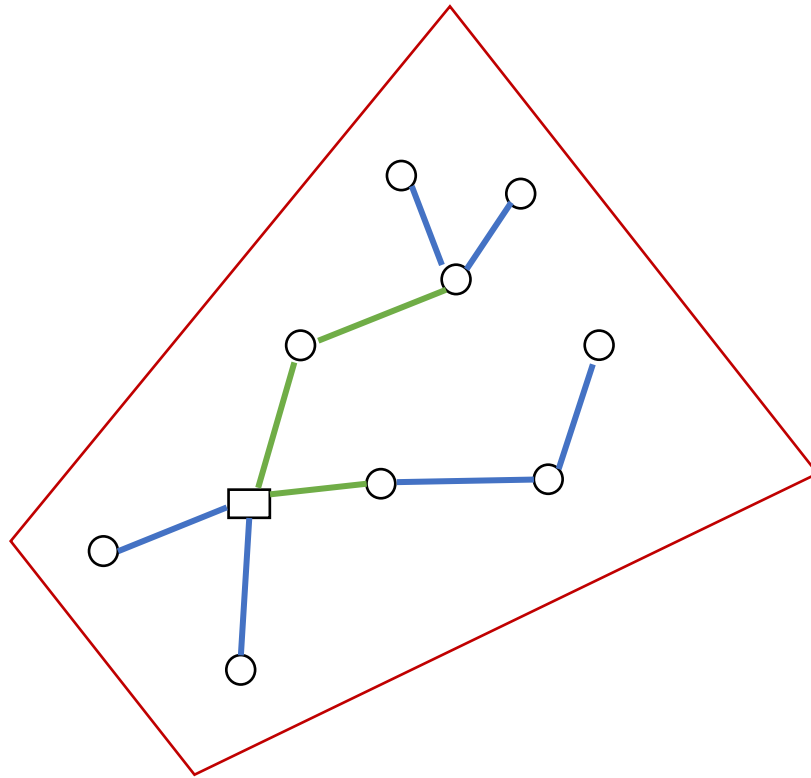
*M. Fischetti, J. R. Kristoffersen, T. Hjort, M. Monaci, and D. Pisinger. Vattenfall optimizes offshore wind farm design. INFORMS Journal on Applied Analytics (2019)*



# Cable routing optimization







*Fischetti, M., Pisinger, D. Optimizing wind farm cable routing considering power losses. European Journal of Operational Research 270(3), 917-930 (2018)*

# Results

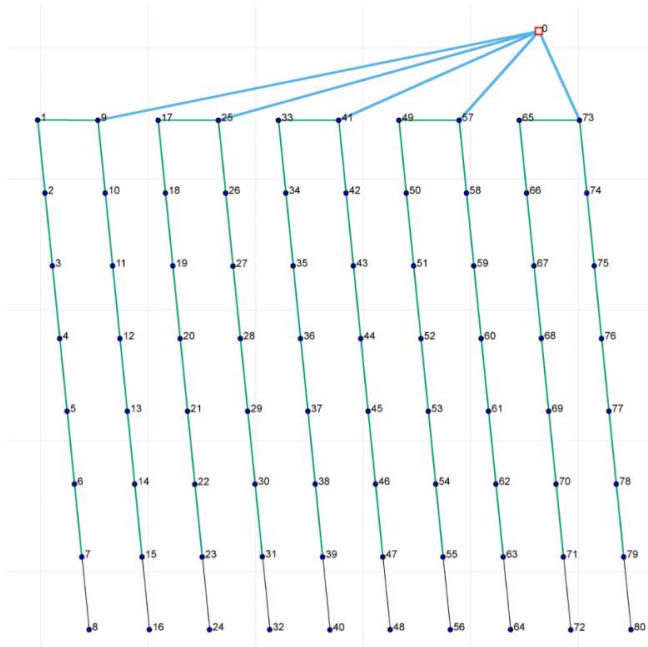


Figure: Existing cable routing for the wind farm Horns Rev 1 in Denmark

## Cable data:

- Black: supports 1 turbine, 345€ /m (CAPEX)
- Green: supports 8 turbines, 385€ /m (CAPEX)
- Blue: supports 16 turbines, 500€ /m (CAPEX)

# Results

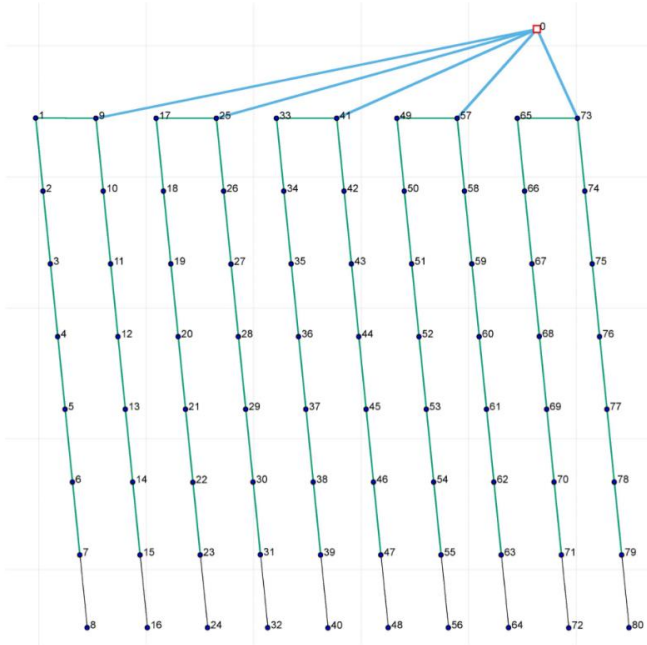
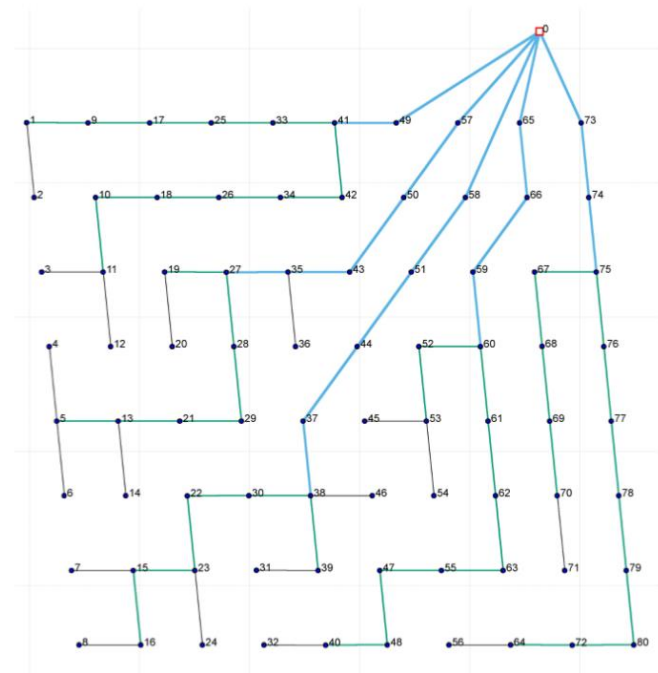


Figure: Existing cable routing for the wind farm Horns Rev 1 in Denmark



Optimized:  
- 1.5 M€

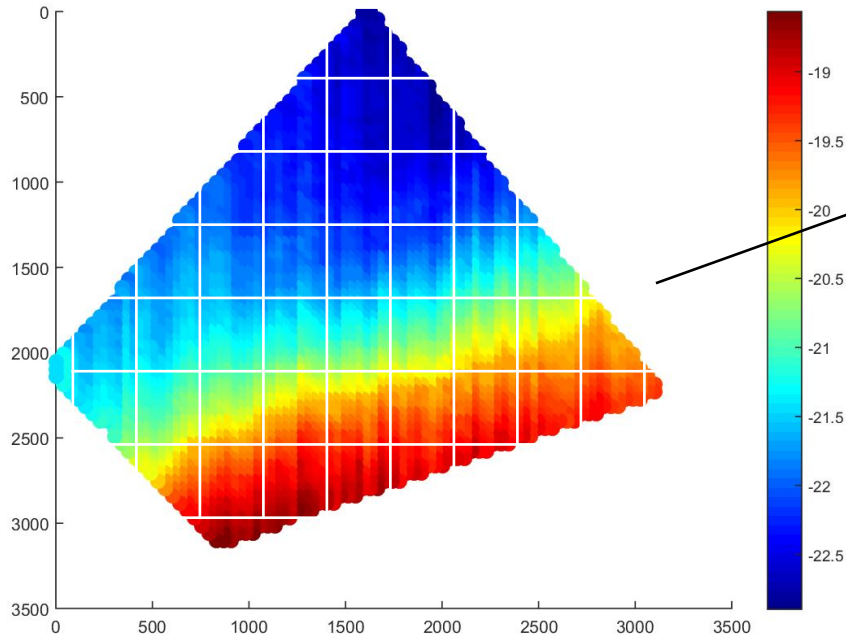


*Thanks*



# Extra

# Mathematical model



The layout problem can be formulated as a MIP problem.  
Variables:

$$x_i = \begin{cases} 1 & \text{if a turbine is built at position } i \in V; \\ 0 & \text{otherwise} \end{cases} \quad (i \in V)$$

where  $V$  is the set of potential turbine positions.

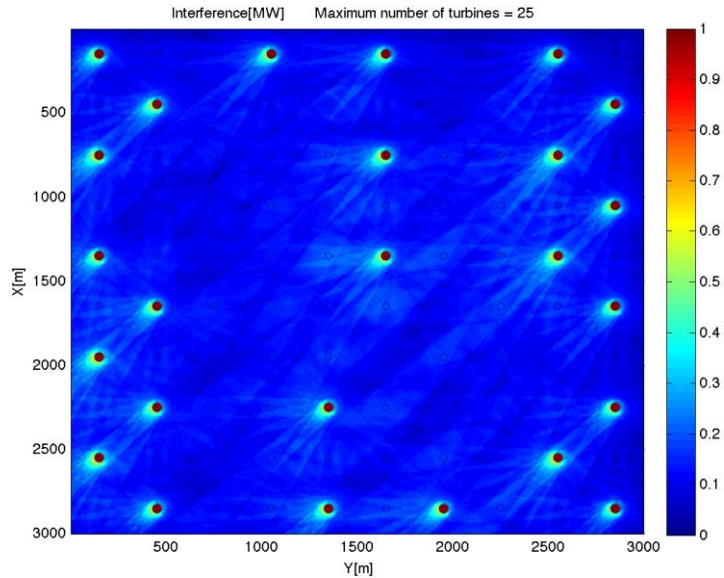
# Mathematical model

$$\begin{aligned} \max \quad & \sum_{i \in V} [(P_i - \frac{c_i}{K_{euro}})x_i - w_i] \\ \text{s.t.} \quad & N_{MIN} \leq \sum_{i \in V} x_i \leq N_{MAX} \\ & x_i + x_j \leq 1 \quad \forall \text{ incompatible } i, j \in V, i < j \\ & \sum_{j \in V} I_{ij}x_j \leq w_i + M_i(1 - x_i) \quad \forall i \in V \\ & x_i \in \{0, 1\} \quad \forall i \in V \\ & w_i \geq 0 \quad \forall i \in V \end{aligned}$$

where  $M_i \gg 0$  (big-M).

$$w_i := \left( \sum_{j \in V} I_{ij}x_j \right) x_i = \begin{cases} \sum_{j \in V} I_{ij}x_j & \text{if } x_i = 1; \\ 0 & \text{if } x_i = 0 \end{cases}$$

# Stochastic programming



It is important to consider wind variability!



Hundred thousands wind scenarios in practice

# Layout optimization - matheuristics

## Why?

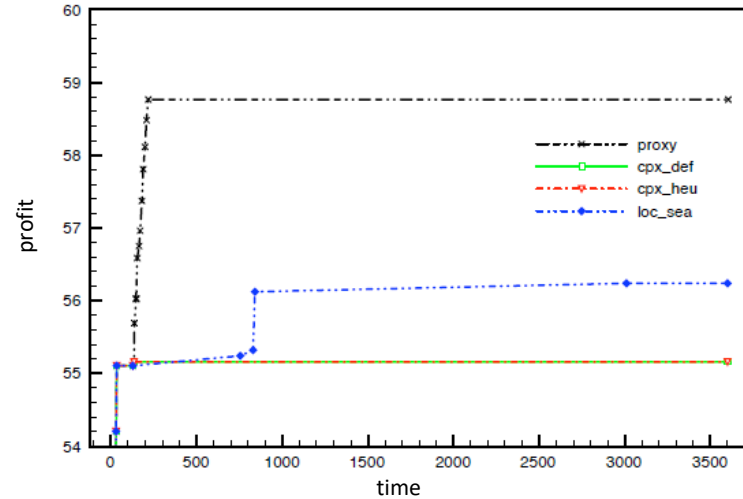
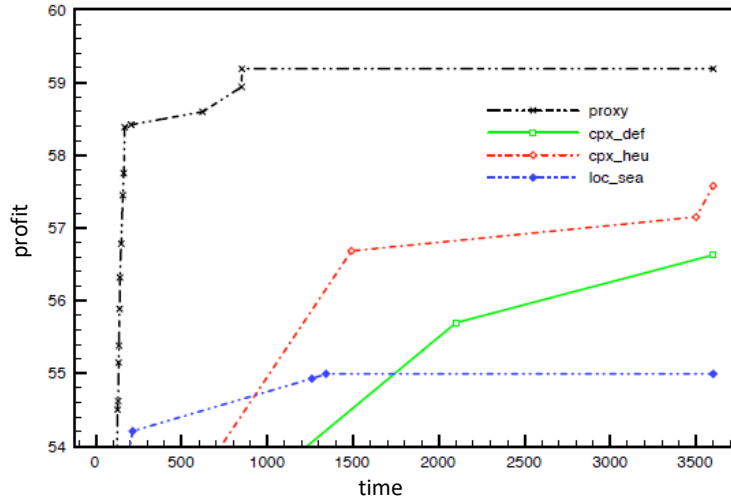
- To solve instances with 20,000+ possible positions

## How?

- Ad-hoc heuristics
- MILP-based heuristics (Proximity Search)

**Idea:** Given an initial (heuristic) solution, explore its neighbourhood using the MILP solver as a black box

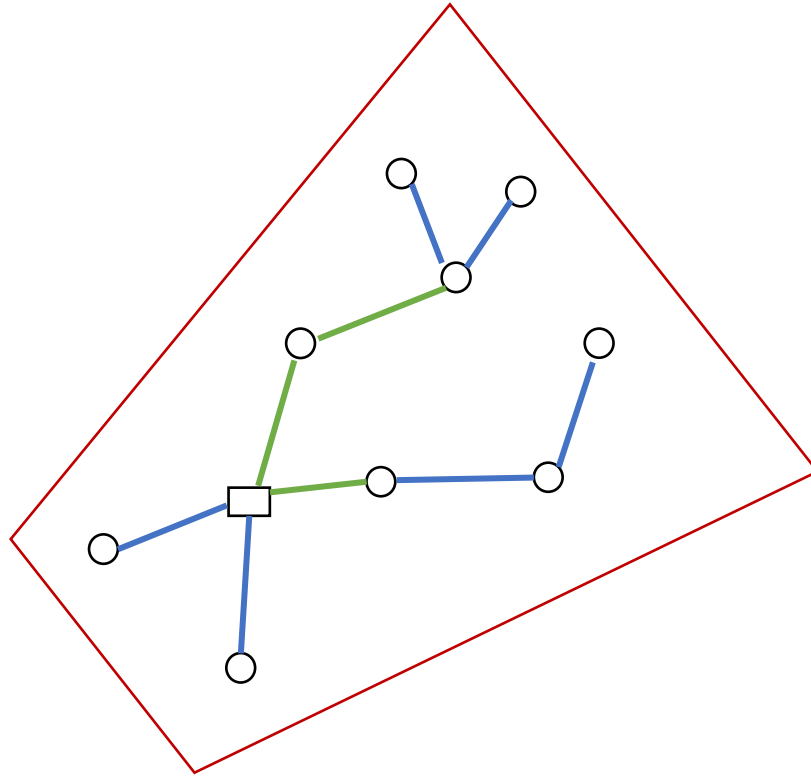
# Layout optimization - matheuristics



Solution profit over time for 2 sample instances with 5000 (left), and 10,000 (right) possible positions; the higher the profit the better.

*M. Fischetti and M. Monaci (2016). Proximity search heuristics for wind farm optimal layout. Journal of Heuristics 22 (4), pp. 459-474.*

# Mathematical model



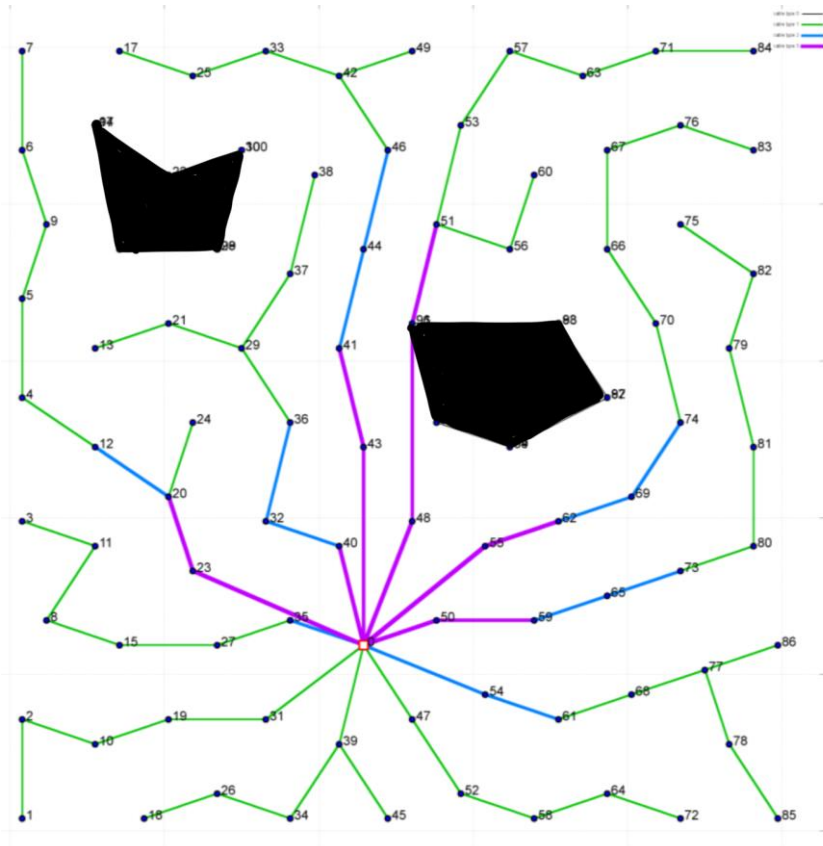
VARIABLES (on the arcs):

$f_{i,j} \geq 0$  is the flow (current) from  $i$  to  $j$

$$x_{i,j}^t = \begin{cases} 1 & \text{if arc } (i,j) \text{ is built with cable } t \\ 0 & \text{otherwise} \end{cases}$$

$y_{i,j} = 1$  if an arc from  $i$  to  $j$  is built,  
with any type of cable ( $\sum_{t \in T} x_{ij}^t = y_{ij}$ )

# Cable routing - MILP formulation



## Limitations:

- Only one cable can exit a turbine
- Capacity
- **No cable-crossings**
- Obstacles
- Maximum number of connections to the substation(s)
- Cable losses



# Cable routing - MILP formulation

$$\begin{aligned}
 \min \quad & \sum_{i,j \in V} \sum_{t \in T} c_{i,j}^t x_{i,j}^t \\
 \text{s.t.} \quad & \sum_{t \in T} x_{i,j}^t = y_{i,j} & i, j \in V : j \neq i \\
 & \sum_{i:i \neq h} (f_{h,i} - f_{i,h}) = P_h & h \in V_T \\
 & \sum_{t \in T} k_t x_{i,j}^t \geq f_{i,j} & i, j \in V : j \neq i, \\
 & \sum_{j:j \neq h} y_{h,j} = 1 & h \in V_T \\
 & \sum_{j:j \neq h} y_{h,j} = 0 & h \in V_0 \\
 & \sum_{i:i \neq h} y_{i,h} \leq C & h \in V_0 \\
 & x_{i,j}^t \in \{0, 1\} & i, j \in V, t \in T \\
 & y_{i,j} \in \{0, 1\} & i, j \in V \\
 & f_{i,j} \geq 0 & i, j \in V, j \neq i
 \end{aligned}$$

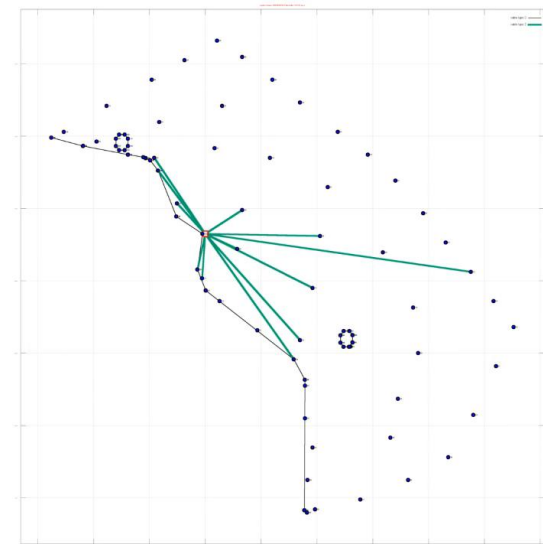
No-crossing constraints are separated on the fly

# Cable routing

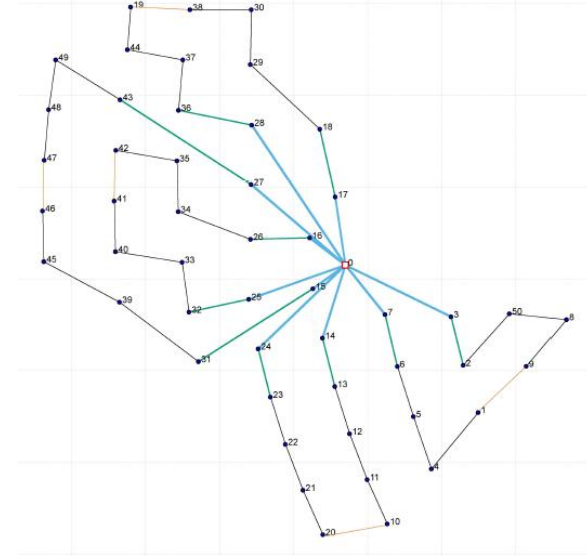
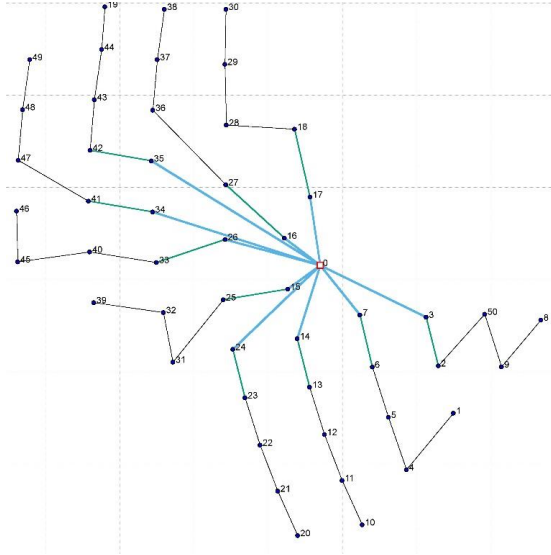
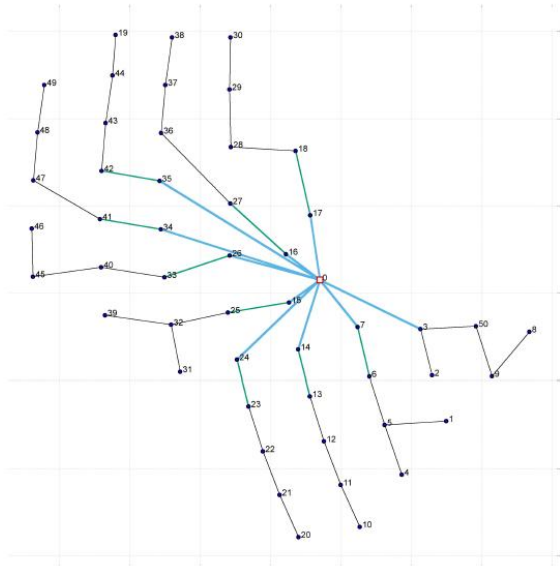
**Problem:** When lots of turbines are involved (or the problem is highly constrained) the model typically requires a long time before producing even the first feasible solution.

We used a **matheuristic approach**:

- First we relax the MILP model (allowing for disconnected solutions)
- Then we solve MILP subproblems to produce a sequence of improved solutions by solving restricted MILPs based on various variable-fixing strategies (random, distance based, by sector)
- Finally, we let the solver run to improve the current solution and (hopefully) solve the problem to proven optimality

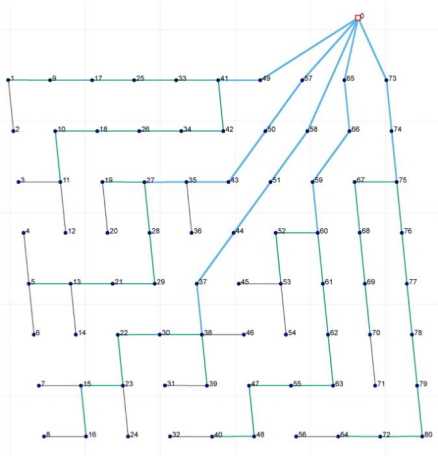


# Cable routing – considering different design options

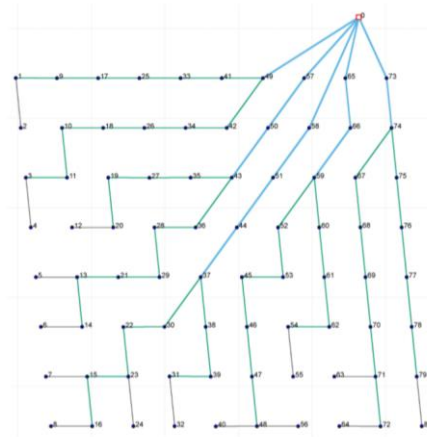


*Fischetti, M., Pisinger, D.: Optimal wind farm cable routing: Modeling branches and offshore transformer modules. Networks 72(1), 42-59 (2018)*

# Results



CAPEX optimized:  
- 1.5 M€ at construction time



CAPEX+losses optimized:  
-1.5 M€ at construction time and  
-1.7 M€ in 25 years